Benchmark Functions for the CEC'2025 Competition on Evolutionary Computation for Consensus-based Distributed Optimization

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1 Introduction

With the rapid development of communication technology, networked systems have emerged in various fields, such as smart grid [1, 2], wireless sensor networks [3, 4], multi-UAV systems (unmanned aerial vehicles) [5, 6]. In networked systems, terminal devices, i.e. nodes, are equipped with basic computing and communication capabilities and connected through a communication network. Nowadays, many researchers focus on studying peer-to-peer communication and cooperation methods for nodes to solve systematic optimization problems, such as electric demand response [7, 8], autonomous target tracking [9, 10], etc. It has the potential to achieve better efficiency, robustness, scalability, and privacy protection than traditional centralized optimization.

Consensus-based distributed optimization (CDO) is a common problem definition for optimization problems in networked systems [11]. In CDO, there is a local objective function for each node, and the systematic objective function, i.e. global objective function, is the sum of all local objective functions. CDO aims to minimize the global objective function and make the nodes reach a consensus on the final solution. There are two major features of CDO, limited local information and no-center local communication. First, each node can only access its own local objective function. This is because the local objective function is usually related to the privacy data, real-time data, or mass data stored in terminal devices, which cannot be transmitted to other nodes. What's more, the local objective functions of different nodes are usually conflicting to some extent. Second, the communication network of CDO is usually a not-fully-connected graph without a center node. Nodes can only communicate with immediate neighbors in the graph. This feature increases the difficulty of fast information transfer among nodes.

In the literature on consensus-based distributed optimization, lots of first-order algorithms have been proposed based on consensus theory and gradient descent method [12, 13]. For example, Nedic and Ozdaglar proposed a distributed subgradient descent method [13]. In addition, zero-order algorithms also attract attention from researchers. Two representative classes are randomized gradient-free algorithms (RGFs) and distributed evolutionary algorithms. RGFs replace the gradient computation of gradient-based algorithms by the differential gradient estimation [14, 15, 16]. Distributed evolutionary algorithms usually create a subpopulation in each agent and evolve subpopulations to optimize the global objective cooperatively [7, 17, 18, 19].

Based on existing literature, black-box CDO still remains promising and challenging. To this end, we design a set of benchmark functions for black-box consensus-based distributed optimization. This benchmark set considers the communication environments, conflict degrees, and node homogeneity. Besides, we provide an algorithm implementation framework, which provides peer-to-peer communication interfaces and performance evaluation for competitors. The benchmark functions and algorithm framework are open-source in the following link:

https://github.com/iamrice/Proposal-for-Competition-on-Black-box-Consensus-based-Distributed-Optimization

In the following, Section 2 introduces the detailed definition of benchmark functions. Section 3 elaborates elementary functions. Finally, the competition protocol is introduced in Section 4.

2 Benchmark functions of consensus-based distributed optimization problems

The problem definition of consensus-based distributed optimization consists of two parts, i.e., objective function and communication network.

2.1 Objective function

For a networked system with n nodes, consensus-based distributed optimization is to minimize the global (systematic) objective function F:

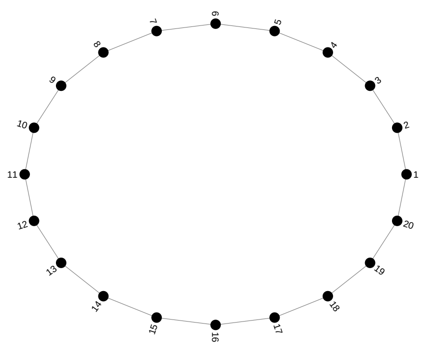
$$\begin{aligned} &\min F(x) = \frac{1}{n}\sum_{i=1}^{n} f_i(x) \\ &\text{where} \quad f_i(x) = f^i_{elementary}(x) + \sigma \sum_{j=1}^{D} [A]_{ij} x_j, \quad i = 1, 2, ..., n \\ &\sum_{i=1}^{n} [A]_{ij} = 0, j = 1, 2, ..., D \end{aligned} \tag{1}$$

Here, f_i is the local objective function of the i-th node, consisting of an elementary function and a linear term. $A \in R^{n\times D}$ is a matrix consisting of n weight vectors for linear terms, and $\sigma \in R$ is the conflict degree. Because the sum of each row in A is 0, the linear items of local objective functions cancel each other out. As a result, the global objective function is reduced to $F(x) = \frac{1}{n}\sum_{i=1}^{n} f_i(x) = \frac{1}{n}\sum_{i=1}^{n} f^i_{elementary}$.

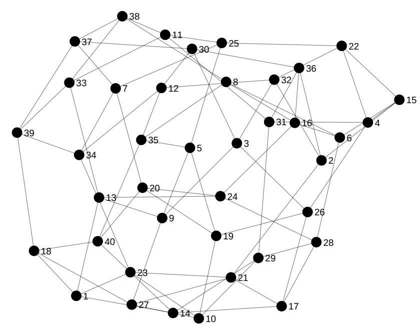
We provide four types of elementary functions, including rotated Elliptic, rotated Rosenbrock, rotated Schewefel, rotated Griewank. They are introduced in Sec. 2.4 in detail.

2.2 Communication network

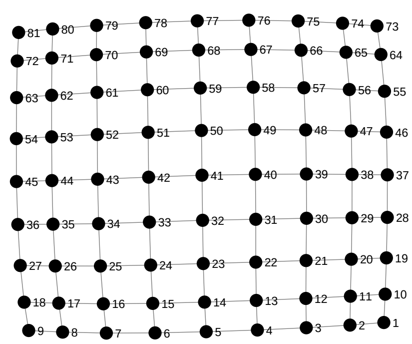
In consensus-based distributed optimization, each node can only communicate with its immediate neighbors. Communication networks affect the problem difficulty and consensus rate. We design four types of communication networks, i.e., 20-nodes ring network, 40-nodes random network, 81-nodes grid network, and a hidden network during competition. They are shown in Fig. 1.



(a) 20-nodes ring network



(b) 40-nodes random network



(c) 81-nodes grid network

Figure 1: Four types of communication networks

2.3 Function settings

Based on the above problem definitions, we design 36 functions from four aspects, i.e., elementary function, conflict degree, communication network, and homogeneity. The settings of G1, G2, G3, G4 are public, while G5 is hidden for test functions. The configuration of benchmark functions is shown in Table 1.

Note that, in heterogeneous functions, the selection of elementary function is configured based on the index of nodes. For example, in the heterogeneous problem "F33" consisted of elliptic and

Table 1: detailed setting of benchmark functions

Group	Index	Elementary functions	Conflict degree	Communication network	Homogeneity
G1	F1	Elliptic	10	20-nodes ring network	Homogeneous
	F2	Rosenbrock			
	F3	Schewefel			
	F4	Griewank			
	F5	Elliptic	100		
	F6	Rosenbrock			
	F7	Schewefel			
	F8	Griewank			
G2	F9	Elliptic	10	40-nodes random network	
	F10	Rosenbrock			
	F11	Schewefel			
	F12	Griewank			
	F13	Elliptic	100		
	F14	Rosenbrock			
	F15	Schewefel			
	F16	Griewank			
G3	F17	Elliptic	10	81-nodes grid network	
	F18	Rosenbrock			
	F19	Schewefel			
	F20	Griewank			
	F21	Elliptic	100		
	F22	Rosenbrock			
	F23	Schewefel			
	F24	Griewank			
G4	F25	Elliptic + rosenbrock	10	40-nodes random network	Heterogeneous
	F26	Elliptic + schwefel			
	F27	Rosenbrock + Schwefel			
	F28	Rosenbrock + griewank			
G5	F29	(hidden for test functions)			
	F30				
	F31				
	F32				
	F33				
	F34				
	F35				
	F36				

rosenbrock, the elementary function of the i-th node is defined as follows:

$$f^{i}_{elementary} = \begin{cases} f_{\text{rotated elliptic}} & i \text{ is odd} \\ f_{\text{rotated rosenbrock}} & i \text{ is even} \end{cases} \tag{2}$$

2.4 Elementary functions

2.4.1 Symbols

- D: problem dimension
- x_{opt}: optimal solution
- R: an orthogonal matrix to make all the variables in the objective function interdependent on each other, ensuring the local objective function is non-separable
- T_{asy}: a transformation function to break the symmetry of the symmetric functions.

$$T_{\text{asy}}^{\beta} : \mathbb{R}^D \to \mathbb{R}^D, x_i \mapsto \begin{cases} x_i^{1+\beta \frac{i-1}{D-1}\sqrt{x_i}} & \text{if } x_i > 0 \\ x_i & \text{otherwise} \end{cases}, \text{for } i = 1, \ldots, D.$$

- T_{osz}: a transformation function to create smooth local irregularities.

$$T_{\text{osz}} : \mathbb{R}^D \to \mathbb{R}^D, x_i \mapsto \text{sign}(x_i)\exp(\hat{x}_i + 0.049(\sin(c_1\hat{x}_i) + \sin(c_2\hat{x}_i))), \text{ for } i = 1, \ldots, D$$

$$\text{where } \hat{x}_i = \begin{cases} \log(|x_i|) & \text{if } x_i \neq 0 \\ 0 \text{ otherwise} \end{cases}, \text{ sgin}(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

$$c_1 = \begin{cases} 10 & \text{if } x_i > 0 \\ 5.5 & \text{otherwise} \end{cases}, \text{and } c_2 = \begin{cases} 7.9 & \text{if } x_i > 0 \\ 3.1 & \text{otherwise.} \end{cases}$$

2.4.2 Rotated Elliptic

$$\begin{aligned} f(x) &= \sum_{i=1}^{D} 10^{6\frac{i-1}{D-1}} z_i^2 \\ \text{where} \quad z &= T_{osz}(Ry) \\ y &= x - x^{opt} \\ x &\in [-100, 100]^D \end{aligned} \tag{3}$$

Properties:

- Unseparable
- Shifted
- Smooth local irregularities

2.4.3 Rotated Schwefel

$$\begin{aligned} f(x) &= \sum_{i=1}^{D}(\sum_{j=1}^{i} z_i)^2 \\ \text{where} \quad z &= T_{asy}^{0.2}(T_{osz}(Ry)) \\ y &= x - x^{opt} \\ x &\in [-100, 100]^D \end{aligned} \tag{4}$$

Properties:

- Unseparable
- Shifted
- Smooth local irregularities
- asymmetric

2.4.4 Rotated Rosenbrock

$$
\begin{aligned}
f(x) &= \sum_{i=1}^{D-1} 100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2 \\
\text{where} \quad z &= Ry \\
y &= x - x^{opt} \\
x &\in [-100, 100]^D
\end{aligned} \tag{5}
$$

Properties:

- Unseparable
- Shifted
- Multimodal

2.4.5 Rotated Griewank

$$
\begin{aligned}
f(x) &= \sum_{i=1}^{D} \frac{x_i^2}{4000} - \prod_{i=1}^{D} cos(\frac{x_i}{\sqrt{i}}) + 1 \\
\text{where} \quad z &= Ry \\
y &= x - x^{opt} \\
x &\in [-100, 100]^D
\end{aligned} \tag{6}
$$

Properties:

- Unseparable
- Shifted
- Multimodal

3 A Real-world problem: multi-target localization problems in wireless sensor networks

Consider a wireless sensor network with n sensors, whose positions are denoted by $y_1, y_2, ..., y_n$. Suppose there are N_t targets appear concurrently in the detection area. Each sensor can receive the signal strength from multiple targets. For sensor i, the measurement data is denoted by $\phi_{i,1}, \phi_{i,2}, ..., \phi_{i,N_t}$.

$$\phi_{i,t} = P_0 - 10n_p \log_{10}(\frac{\|p_t - y_i\|}{d_0}) + \epsilon \tag{7}$$

where $\epsilon \sim \mathcal{N}(0, v)$ is the measurement noise. P_0 is a known reference power value in dB milliwatts at a reference distance d_0.

A sensor can estimate the distance from targets to itself according to the signal strength. However, due to the lack of direction information, it is hard for a single sensor to locate targets. Therefore, it is necessary for multiple sensors to cooperate and locate targets. As a result, the multi-target localization problem is defined as follows:

$$F(p_1, p_2, ..., p_{N_t}) = \frac{1}{n}\sum_{i=1}^{n} f_i(p_1, p_2, ..., p_{N_t}) \tag{8}$$

$$\text{s.t.} \quad f_i(*) = \sum_{t=1}^{N_t}[\phi_{it} - (P_0 - 10n_p \log_{10}(\frac{\|p_t - y_i\|}{d_0}))]^2$$

Here, $p_1, p_2, ..., p_{N_t}$ are the optimization variables, where $p_t \in R^3$ represents the estimated position of target t. $f_i(*)$ is the local objective function of sensor i, which represents the estimation error based on the measurement data of sensor i. $F(*)$ is the global objective function, which is the total estimation error of all sensors.

4 Competition framework and evaluation procedures

4.1 Implementation framework of distributed algorithms

There are two principles of the algorithm implementation. First, each node can only access its own local objective function f_i. Second, each node can only communicate with its immediate neighbors in the system. The participants need to develop the algorithm in the node-level. This algorithm is applied in all nodes equally.

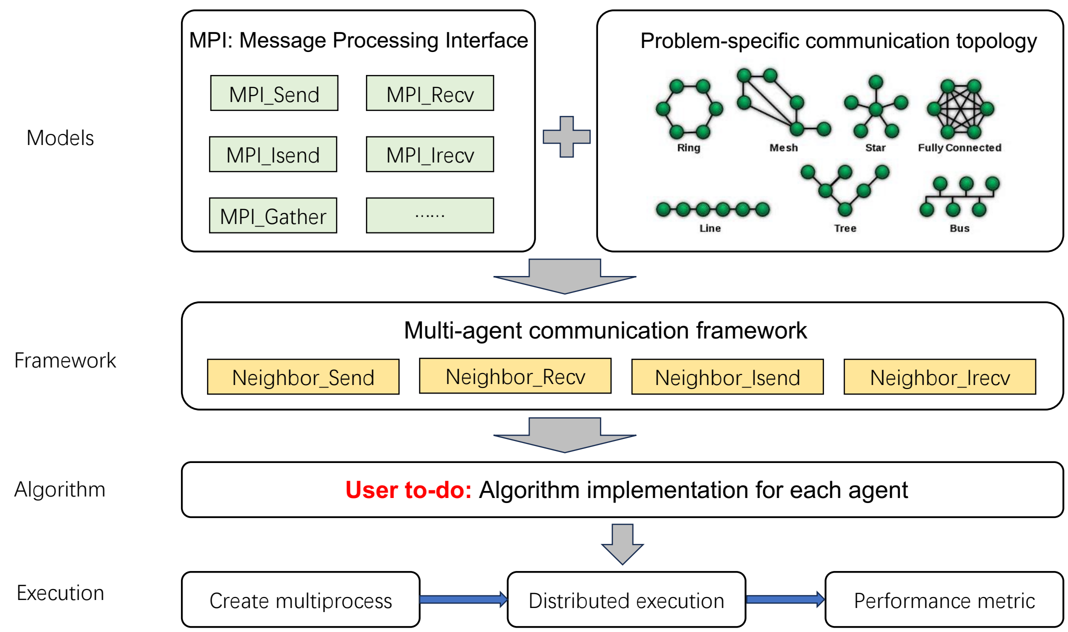


Figure 2: Algorithm framework of the competition

We provide the following interfaces of the framework:

- Interfaces for communication: includes blocking communication and non-blocking communication. We define the range of communication for each node according to the communication network.
- Interface for fitness evaluation: returns fitness of local objective function. Each agent can only call the objective function of itself.
- Interface for performance evaluation: returns the algorithm performance, including fitness, system disagreement, and communication cost.

4.2 Evaluation procedures

When the maximum evaluation number is reached, the framework will collect the final solutions $\{x_1, x_2, ..., x_n\}$ and communication cost $\{c_1, c_2, ..., c_n\}$ of the n nodes. Then, three evaluation criterions are computed as follows.

- Fitness of average final solutions $\mathcal{F}$: $F(\frac{1}{n}\sum_{i=1}^{n} x_i)$, where F is the global objective function.
- System disagreement: $\sum_{i=1}^{n} \|x_i - \overline{x}\|$ where $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$.
- Communication cost $\mathcal{C}$: $\sum_{i=1}^{n} c_i$.

In this competition, it is necessary for all nodes to reach a consensus finally. Therefore, a final solution is valid only when the system disagreement is lower than a threshold $\epsilon = 1e-3$. For valid solutions, we use Z-Score normalization to transform the fitness and the communication cost independently, and add them together as the final score. Suppose there are m competitors with fitness $\{\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_m\}$ and communication cost $\{\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m\}$ after optimizing a benchmark function, the final score of the k-th competitor on this benchmark function is defined as:

$$\frac{\mathcal{F}_k - \mu_{(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_m)}}{\sigma_{(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_m)}} + \frac{\mathcal{C}_k - \mu_{(\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m)}}{\sigma_{(\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m)}}$$

The cumulative sum of scores of all tested functions will be used for ranking the contestants.

4.3 Example of representing the results

Table 2: results record

Group	Index	Disagreement <1e-3		Group	Index	Disagreement <1e-3	
		Fitness	Communication cost			Fitness	Communication cost
G1	F1			G3	F17		
	F2				F18		
	F3				F19		
	F4				F20		
	F5				F21		
	F6				F22		
	F7				F23		
	F8				F24		
G2	F9			G4	F25		
	F10				F26		
	F11				F27		
	F12				F28		
	F13				F29		
	F14				F30		
	F15				F31		
	F16				F32		
G5	F33						
	F34						
	F35						
	F36						

5 Biography of organizers

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References

[1] M. Papadimitrakis, N. Giamarelos, M. Stogiannos, E. Zois, N.-I. Livanos, and A. Alexandridis, "Metaheuristic search in smart grid: A review with emphasis on planning, scheduling and power flow optimization applications," vol. 145, p. 111072.

[2] D. K. Molzahn, F. Dörfler, H. Sandberg, S. H. Low, S. Chakrabarti, R. Baldick, and J. Lavaei, "A survey of distributed optimization and control algorithms for electric power systems," vol. 8, no. 6, pp. 2941–2962. Conference Name: IEEE Transactions on Smart Grid.

[3] P. Dames and V. Kumar, "Autonomous localization of an unknown number of targets without data association using teams of mobile sensors," *IEEE Transactions on Automation Science and Engineering*, vol. 12, no. 3, pp. 850–864. Conference Name: IEEE Transactions on Automation Science and Engineering.

[4] M. F. AbdelHaq and A. Salman, "Wireless sensor network for traffic monitoring," in *2020 International Conference on Promising Electronic Technologies (ICPET)*, pp. 16–21.

[5] Q. Luo and H. Duan, "Distributed UAV flocking control based on homing pigeon hierarchical strategies," vol. 70, pp. 257–264.

[6] Y. Yu, H. Wang, S. Liu, L. Guo, P. L. Yeoh, B. Vucetic, and Y. Li, "Distributed multi-agent target tracking: A nash-combined adaptive differential evolution method for UAV systems," vol. 70, no. 8, pp. 8122–8133. Conference Name: IEEE Transactions on Vehicular Technology.

[7] K. Utkarsh, A. Trivedi, D. Srinivasan, and T. Reindl, "A consensus-based distributed computational intelligence technique for real-time optimal control in smart distribution grids," vol. 1, no. 1, pp. 51–60. Conference Name: IEEE Transactions on Emerging Topics in Computational Intelligence.

[8] P. Faria, J. Soares, Z. Vale, H. Morais, and T. Sousa, "Modified particle swarm optimization applied to integrated demand response and DG resources scheduling," vol. 4, no. 1, pp. 606–616. Conference Name: IEEE Transactions on Smart Grid.

[9] L. Rakai, H. Song, S. Sun, W. Zhang, and Y. Yang, "Data association in multiple object tracking: A survey of recent techniques," *Expert Systems with Applications*, vol. 192, p. 116300.

[10] A. T. Kamal, J. H. Bappy, J. A. Farrell, and A. K. Roy-Chowdhury, "Distributed multi-target tracking and data association in vision networks," vol. 38, no. 7, pp. 1397–1410. Conference Name: IEEE Transactions on Pattern Analysis and Machine Intelligence.

[11] T. Yang, X. Yi, J. Wu, Y. Yuan, D. Wu, Z. Meng, Y. Hong, H. Wang, Z. Lin, and K. H. Johansson, "A survey of distributed optimization," vol. 47, pp. 278–305.

[12] A. Nedic, A. Olshevsky, and W. Shi, "Achieving geometric convergence for distributed optimization over time-varying graphs," vol. 27.

[13] A. Nedic and A. Ozdaglar, "Distributed subgradient methods for multi-agent optimization," vol. 54, no. 1, pp. 48–61.

[14] D. Yuan and D. W. C. Ho, "Randomized gradient-free method for multiagent optimization over time-varying networks," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 26, pp. 1342–1347, June 2015.

[15] M. Chamanbaz, G. Notarstefano, F. Sasso, and R. Bouffanais, "Randomized constraints consensus for distributed robust mixed-integer programming," *IEEE Transactions on Control of Network Systems*, vol. 8, pp. 295–306, Mar. 2021.

[16] Y. Pang and G. Hu, "Randomized gradient-free distributed optimization methods for a multiagent system with unknown cost function," *IEEE Transactions on Automatic Control*, vol. 65, no. 1, pp. 333–340. Conference Name: IEEE Transactions on Automatic Control.

[17] W. Ai, W. Chen, and J. Xie, "A general framework for population-based distributed optimization over networks," *Information Sciences*, vol. 418–419, pp. 136–152, Dec. 2017.

[18] Y. Wakasa and S. Nakaya, "Distributed particle swarm optimization using an average consensus algorithm," in *2015 54th IEEE Conference on Decision and Control (CDC)*, pp. 2661–2666, 2015.

[19] M. K. Jalloul and M. A. Al-Alaoui, "A distributed particle swarm optimization algorithm for block motion estimation using the strategies of diffusion adaptation," in *2015 International Symposium on Signals, Circuits and Systems (ISSCS)*, pp. 1–4, July 2015.